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Beamforming Technique for Investigation of Lateral Variability at Geotechnical Sites

(Teknik Alur Bentuk untuk Mengkaji Kebolehubahan Sisi di Tapak Geoteknik)

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ABSTRACT

A beamformer in seismology is a signal receptor with a series of geophones, in which a beam of elastic waves is formed like a light beam by adjusting signal delays at individual geophones. Recently, beamforming has extended its applications to surface-wave measurement. In surface-wave measurement, beamforming provides unique advantages over other surface-wave methods, such as full automation in data analysis as well as directional signal reception to minimize scattered noise and multiple reflections in signals. However, certain defects depreciate the value of beamforming in terms of its practicality and feasibility. These include the requirement of having many receivers and the loss of small wavelength data due to spatial aliasing. It leads to insensitivity in identification of lateral variability, which creates the problem of having to smooth out geologic features and complexities like folding, faults and fractures. In this paper, advances in the refinement of beamforming were described on two counts: improvement of sensitivity in identification of lateral variability and recovery of aliased wave numbers, which enables evaluation of shallow material. On the passage to refinement, synthetic waveforms for typical layering systems were generated to figure out characteristics of beamformer velocities in comparison with SASW velocities and theoretical normal-mode velocities.

Keywords: Beamforming theory; phase velocities; refined beamformer; spatial aliasing

ABSTRAK

Alur bentuk dalam konteks seismologi ialah pengesan isyarat yang menggunakan sebilangan geofon, dengan alur gelombang kenyal seperti alur cahaya dibentuk dengan mengubah lengah isyarat pada setiap geofon. Kebelakangan ini, aplikasi alur bentuk telah dikembangkan dalam kaedah pengukuran gelombang permukaan. Dalam kaedah pengukuran gelombang permukaan, alur bentuk mempunyai kelebihan yang unik berbanding kaedah gelombang permukaan yang lain, seperti automasi penuh dalam menganalisis data serta penerimaan isyarat terarah untuk meminimumkan taburan hingar dan pantulan berganda dalam isyarat. Walau bagaimanapun, terdapat beberapa kekangan yang boleh menurunkan nilai alur bentuk daripada segi praktikal dan pelaksanaan. Ia termasuklah keperluan untuk menggunakan banyak penerima dan kehilangan data pada panjang gelombang yang pendek akibat herotan ruang. Ini membuatkan tahap pengenalpastian terhadap kebolehubahan sisi menjadi kurang sensitif dan boleh menimbulkan masalah kerana terpaksa melicinkan fitur geologi serta struktur rencamnya seperti lipatan, sesaran dan retakan. Dalam kertas ini, kemajuan dalam tapisan alur bentuk diterangkan berdasarkan dua penekanan: penambahbaikan kepekaan dalam pengenalpastian kebolehubahan sisi dan pemulihan nombor gelombang yang terherot, bagi membolehkan penilaian dilakukan ke atas bahan cetek. Dalam proses penapisan ini, sejumlah gelombang sintetik untuk sistem berlapis yang tipikal telah dihasilkan bagi mencari dan menentukan ciri-ciri halaju alur bentuk, berdasarkan perbandingan dengan halaju SASW dan halaju teori mod-normal.

Kata kunci: Alur bentuk tertapis; halaju fasa; herotan ruang; teori beamforming

INTRODUCTION

Site investigation for layer stratification or stiffness profile is not the only use of surface-wave measurement. Successful structural integrity assessment for both concrete and geotechnical structures are also attributed to the advancement of surface-wave technique. With an increasing demand for surface-wave measurement, its technology is also moving from 1-D profiling to 3-D representation, from expertise-intensive analysis to fully automated analysis.

Currently, a variety of surface-wave assessment methods are available such as SASW, MASW, CSW, ReMi and frequency-wave number technique (Joh 1996; Stokoe et al. 1994, 2004). Each of these surface-wave methods has its own pros and cons. In most cases, the accuracy and reliability do not go along with practicality and feasibility in testing and analysis. Recently, a new method called beamforming technique, which has been widely used for radar, sonar and wireless transmission, was also adopted for evaluating shear-wave velocity profiles at geotechnical sites. Thanks to several advantages of beamforming technique, such as full automation in data analysis and minimization of noises and multiple reflections, beamforming could be a leading technology in surface-wave measurement. However, beamformer faces a couple of limitations in practice, because a long measurement array smoothens out important geologic features locally distributed and also because large wave numbers are aliased, which precludes the possibility of evaluating shallow material.

To refine the beamforming technique for improved performance, two elementary techniques were proposed in this paper: four-geophone based walk-away procedure and recovery scheme of aliased wave numbers. To verify the validity of the refined beamformer, synthetic waveforms for a typical layering system were incorporated.

BEAMFORMING THEORY

A beamforming technique was developed for transmitting and receiving signals in a specific direction. Beamforming in signal reception implies that a beam of elastic waves is formed like a light beam by adjusting signal delays corresponding to individual receivers. Delay-and-sum beamforming is the oldest and simplest beamforming algorithm. In the delay-and-sum beamforming, if a seismic wave is propagating towards a receiver array, the receiver outputs, delayed by appropriate amounts and added together, reinforce the seismic signal with respect to noise or waves propagating in different directions (Johnson & Dedgeon 1993). Let the waveform measured by the m^{th} receiver $h_m(t)$. Delay-and-sum beamformer consists of applying a delay Δ_m and weighting factor w_m to the amplitude of each receiver, then summing the resulting signals. The delays are chosen to maximize the array's sensitivity to waves propagating from a particular location in space. The output signal from the delay-and-sum beamformer is defined as:

$$z(t) = \sum_{m=1}^{M} w_m h_m \left(t - \Delta_m\right). \tag{1}$$

The beamforming algorithm in the time domain can be also expressed in the frequency domain. Phase theorem, a property of Fourier transform, allows time delays to be rephrased in phase shift. Therefore (1) can be redefined as (2) using $H_m(\omega)$, which denotes Fourier transform of a waveform at the mth receiver.

$$Z(\omega) = \sum_{m=1}^{M} w_m H_m(\omega) e^{-j\omega\Delta_m}. \tag{2}$$

Since phase delay $\omega\Delta_m$ can be replaced with $-k \cdot x_m$, which is a product of local wavenumber k and the location of the m^{th} receiver location, (2) can be simplified as (3),

$$Z_n = \mathbf{e}_n \mathbf{H}_n, \tag{3}$$

where $\mathbf{e}_n$ is defined as (4) and called the steering vector for the n^{th} frequency bin, a sequence of phasors whose exponents work to cancel phase shift corresponding to the waveform of the m^{th} receiver.

$$\mathbf{e}_n = \left\{ w_1 e^{-jkx_1}, w_2 e^{-jkx_2}, \ldots, w_m e^{-jkx_m}, \ldots, w_M e^{-jkx_M} \right\}^T, \tag{4}$$

$\mathbf{H}_n$ is a column vector weighted by w_m that contains the FFT coefficients for the waveform from each receiver. Finally, the steered response power for the n^{th} frequency bin, which indicates the power in the beamformer's output spectrum, can be defined as (5),

$$P_n = \mathbf{e}_n E[\mathbf{H}_n \mathbf{H}_n] \mathbf{e}_n = \mathbf{e}_n \mathbf{R}_n \mathbf{e}_n, \tag{5}$$

where the matrix $\mathbf{R}_n$ is known as a cross-spectral matrix or a spatial correlation matrix.

Beamforming technique has been successfully employed for surface-wave measurements at geotechnical sites by researchers (Zywicki & Malladi 2007; Zywicki & Rix 2005). However, to make beamforming more practical, some improvements need to be made to deal with its limitations, namely, large number of receivers required to produce a good resolution and its lack of high-frequency information due to spatial aliasing. At first, the requirement of many receivers leads to a couple of problems, including insensitivity in lateral variability due to long measurement array and high standard in hardware specification. Secondly, in the case of spatial aliasing, wave numbers can be aliased if wave number goes over $2\pi/d$, where d is inter-receiver spacing. Therefore, high frequencies to cause aliasing in wave numbers (spatial aliasing) cannot be included in surface-wave dispersion curves. In turn, the dispersion curve lacks the short wavelengths required to evaluate stiffness of shallow material. In this research two elementary techniques for beamforming were proposed to enhance its performance in the area of surface-wave measurements.

WALK-AWAY APPROACH FOR LIMITED NUMBER OF RECEIVERS

In practicing beamforming technique, using many receivers is preferable for reliable determination of phase velocities. One example of the technique is to incorporate 15 geophones for characterizing geotechnical sites (Zywicki & Rix 2005). Besides hardware requirements for having a number of receivers, long measurement array also leads to the problem of smoothing out lateral variability in sedimentary strata such as geological folds and faults.

To solve these problems, a testing scheme called the walk-away test is proposed and adopted. The walk-away test makes repeated measurements with increasing source offsets using a limited number of receivers and then combines all the waveforms together. Then, the total length of the measurement array becomes equivalent to the combined length of the source offset and the actual length of receiver arrays. Therefore, the total number of waveforms increases to much more than the actual number of receivers. We incorporated this idea to apply beamforming technique to the investigation of a limited section with only four receivers. The details of walk-away beamforming measurements are illustrated in Figure 1. In the figure, four independent sources are applied consecutively to a layered system with four receivers deployed at the same spacing. Each source produces four waveforms, which are combined together to come up with 13 waveforms.

Analysis was done by incorporating the signal processing algorithm using MATLAB software (R2010a). The combination of waveforms induced by different sets of sources is based on impulse response, which is defined as system response to an impulse source. That is, waveforms measured at all receivers are processed to determine impulse responses equivalent to the impulse source given at the location of source S1. The impulse responses are then multiplied by a typical source function, i.e. half-sine wave and converted to waveforms. The details of determining impulse responses are as follows. First of all, for each set of sources, transfer functions are computed between receivers 2 to 4 and receiver 1. Receiver 1 is a local reference receiver for computation of transfer functions. Then transfer functions for the global reference, H_{p1}, are recalculated using (6), where the global reference is the first receiver in a measurement array. For example, the transfer function for receiver No. 5 and reference receiver No. 1 can be determined as in (7).

$$H_{p1} = \frac{H_p}{H_r} \times \frac{H_r}{H_1} = H_{pr} \times H_{r1}. \tag{6}$$

$$H_{51} = \frac{H_5}{H_4} \times \frac{H_4}{H_1} = H_{54} \times H_{41} = H_{21.2} \times H_{41.1}. \tag{7}$$

Then, the linear spectrum of a typical source function, S, is multiplied to transfer function as in (8), which results in the transfer function between receiver p and a typical source. Finally, waveforms corresponding to a typical source located at the first receiver are determined by (9).

$$H_{p1,S} = H_{p1} \times S. \tag{8}$$

$$h_{p,S} = \text{IFFTH}(H_{p1,S}). \tag{9}$$

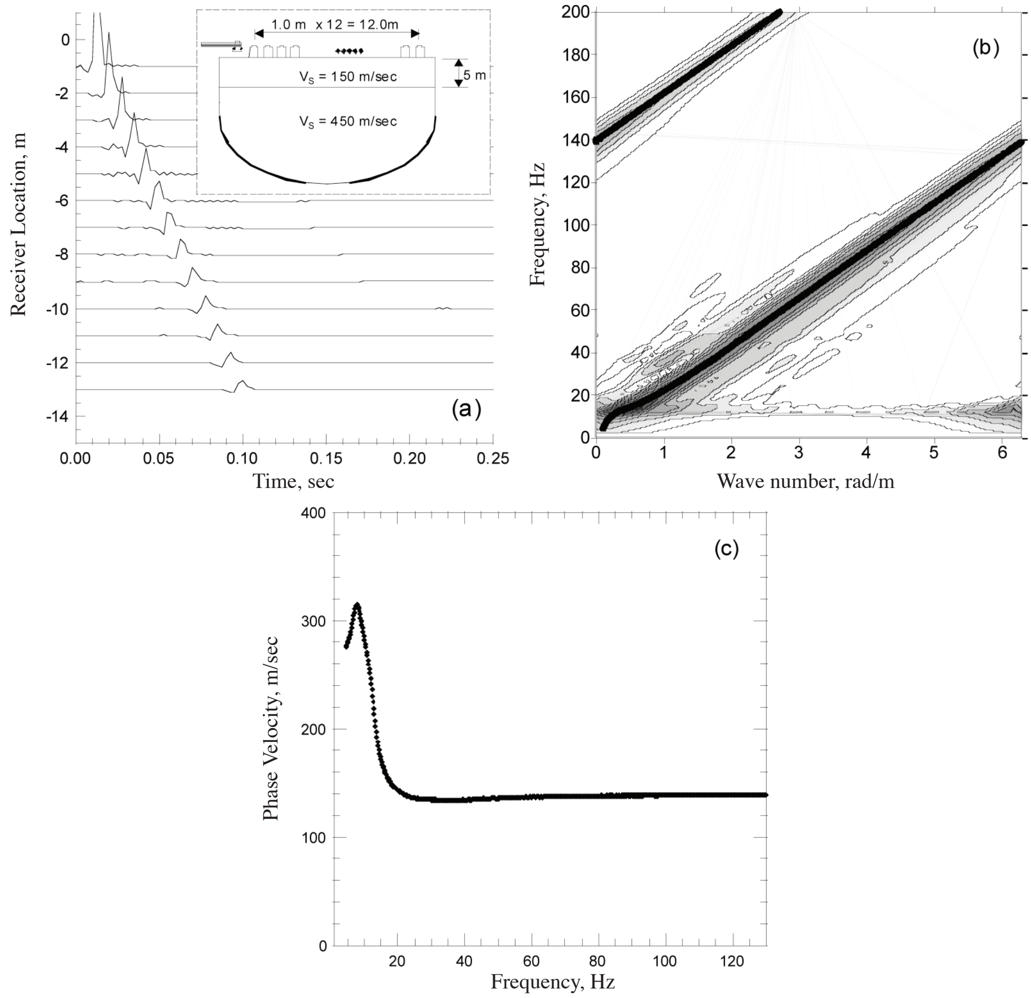

FIGURE 1. Application of beamforming technique to synthetic waveforms determined at a shallow bedrock site: (a) synthetic waveforms, (b) frequency-wavenumber map and (c) phase velocity dispersion curve

RECOVERY OF SPATIAL ALIASING

Beamforming tests have a lower limit in measured wavelengths. The lower limit of a wavelength, that is, minimum wavelength, λ_{min}, is the reciprocal of maximum wave number, k_{max}, as shown in (10). Wave number can be expressed as the gradient of phase angle change, $\Delta\phi$, with respect to distance of wave propagation, r. For two adjacent receivers, the largest phase angle to be measured is 2π due to the features of trigonometric algebra; receiver spacing is d, so that maximum wave number is defined as (11). As a result, minimum wavelength is defined to be the same as receiver spacing (12).

Spatial aliasing is a phenomenon, in which a wave number is decreased by $2\pi/d$ when the wave number exceeds a maximum wave number, k_{max}. This aliasing phenomenon can be described in terms of phase-angle wrapping. In trigonometric algebra, when a phase angle exceeds 2π, trigonometric functions give the same value for the angle subtracted by an angle of $2n\pi$, where n could be 0, 1, 2, or any integer. Therefore, the wave number is limited to the upper limit of k_{max}, which gives rise to the problem of aliasing. An understanding of this problem leads to an idea of how to overcome aliasing and to recover aliased wave number. By replacing $\Delta\phi$ with $\Delta\tilde{\phi} + 2n\pi$ where $\Delta\tilde{\phi}$ is a wrapped phase angle smaller than 2π, we proposed to recover the aliased wave number by (13):

$$\lambda_{min} = \frac{2\pi}{k_{max}}. \quad (10)$$

$$k = \frac{\Delta\phi}{r} \quad \text{i.e.,} \quad k_{max} = \frac{2\pi}{d}. \quad (11)$$

$$\lambda_{min} = d. \quad (12)$$

$$k = \frac{\Delta\phi}{d} = \frac{\Delta\tilde{\phi} + 2n\pi}{d} = \frac{\Delta\tilde{\phi}}{d} + \frac{2n\pi}{d} = \tilde{k} + \frac{2n\pi}{d}n, \quad (n = 0,1,2,\ldots). \quad (13)$$

Illustrations on recovery of aliased wave number are given in Figure 2. For a three-layer system, two sets of 13 waveforms from receivers offset by 1 m and 3 m, respectively, were generated synthetically by dynamic stiffness modeling. Those waveforms were processed by the proposed beamforming algorithm, by using MATLAB software (R2010a) to give frequency-wave number maps as shown in Figure 3. Discontinued parallel gray lines represent aliased wave numbers and dark solid lines stand for recovered wave numbers. Then, recovered wave numbers were used with corresponding frequencies to calculate phase velocities by (14) as shown in Figure 4;

$$v_{ph} = \frac{2\pi f}{k}. \quad (14)$$

For verification, the maximum wave numbers were retrieved from the unaliased part of the curve, denoted as k_{max} in Figure 3, which are 6.28 and 2.09, respectively, for 1 m and 3 m receiver spacing. From the maximum wave numbers, minimum wavelengths are then determined by (10). The resulting minimum wavelengths for 1 m and 3 m receiver spacing are 1.00 m and 3.02 m, respectively. Therefore, it can be seen clearly that the minimum wavelength in beamforming tests is equivalent to receiver spacing, as shown in (12).

FIELD APPLICATIONS

Beamforming technique refined with the walk-away procedure and the spatial-aliasing recovery scheme was

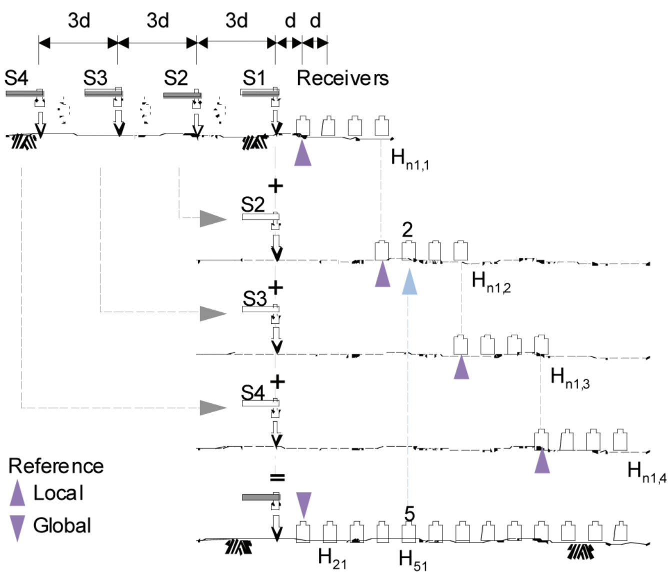

FIGURE 2. Schematic diagram to combine waveforms from a different set of source functions by the proposed walk-away testing method

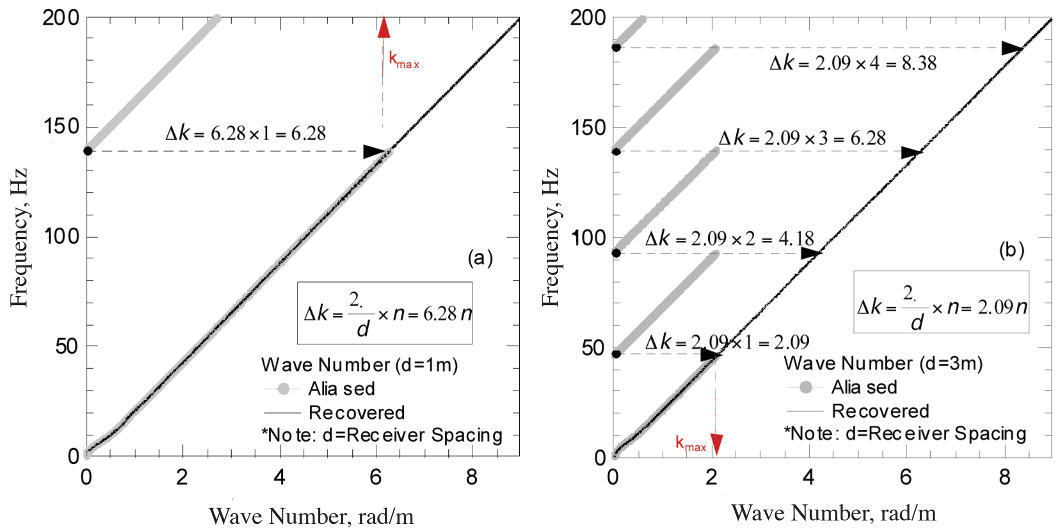

FIGURE 3. Algorithm to recover aliased wave numbers:
(a) receiver spacing = 1 m and (b) receiver spacing = 3 m

applied to measurements of surface-wave velocities at natural geotechnical sites: a natural soil deposit over bedrock at Hornsby, Texas in the United States and an engineering fill over bedrock at DaeGu in Korea.

Surface-wave tests at Hornsby were carried out by the University of Texas at Austin using a vibroseis truck as a source and 1 Hz geophones. Waveforms were recorded at 41 geophones with 5 m receiver spacing and the same set of data was used for SASW analysis and beamforming analysis. The results are shown in Figure 5. The frequency-wave number map indicates that there are several wave groups, making wave propagation complicated. Selection of a governing wave group leads to an almost perfect match between beamforming velocities and SASW velocities. Agreement between two velocities is observed from low to high frequencies. To figure out any higher-mode complexity involved in the measurements, normal-mode velocities were generated for the layering system with the shear-wave velocity profile inverted from SASW measurements. As shown in Figure 5(b), both beamforming velocities and SASW velocities happen to fall on the fundamental mode, which is a typical trend for a soil deposit over a bedrock site.

Comparison between beamforming velocities and SASW velocities was also made at an engineering fill in DaeGu, Korea. Both tests were performed using a 100 kg drop weight and four seismic accelerometers, as proposed previously in walk-away section, with 3

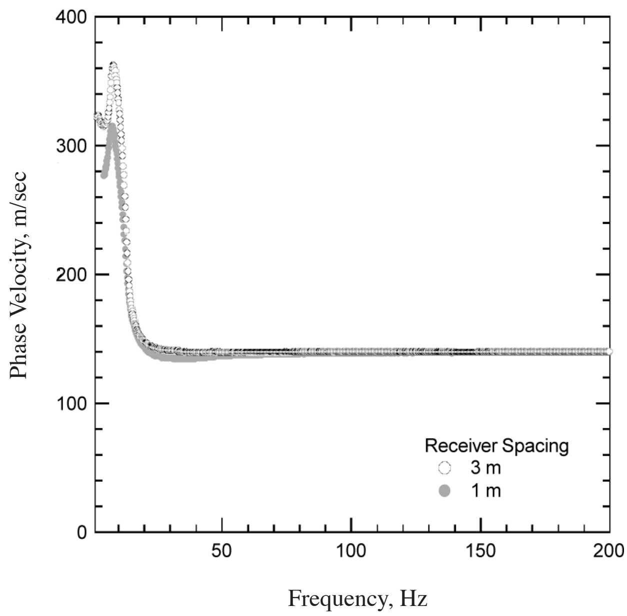

FIGURE 4. Resulting dispersion curves based on recovered
wave numbers in Figure 3

m receiver spacing. At high frequencies over 20 Hz, both velocities are in a good agreement, but at lower frequencies beamforming velocities are evaluated lower than SASW velocities. Judging from the fact that no dominant wave groups are available at frequencies lower than 15 Hz in the f-k map, shown in Figure 6, the source employed for beamforming tests did not generate low-frequency energy sufficient for sampling deep material. Therefore, it is apparent that beamforming tests require more surface-wave energy than SASW tests to determine reliable phase velocities in a low frequency region.

CONCLUSION

The beamforming technique, developed for transmitting and receiving signals in a specific direction has been widely used for radar, sonar and wireless communications. In this paper, beamforming technique was refined for surface-wave velocity measurements, in order to overcome limitations such as insensitivity to lateral variability and spatial aliasing. Additionally, the inherent features of surface-wave velocities determined by beamforming technique were investigated using synthetic waveforms for typical layering systems. The key findings of this

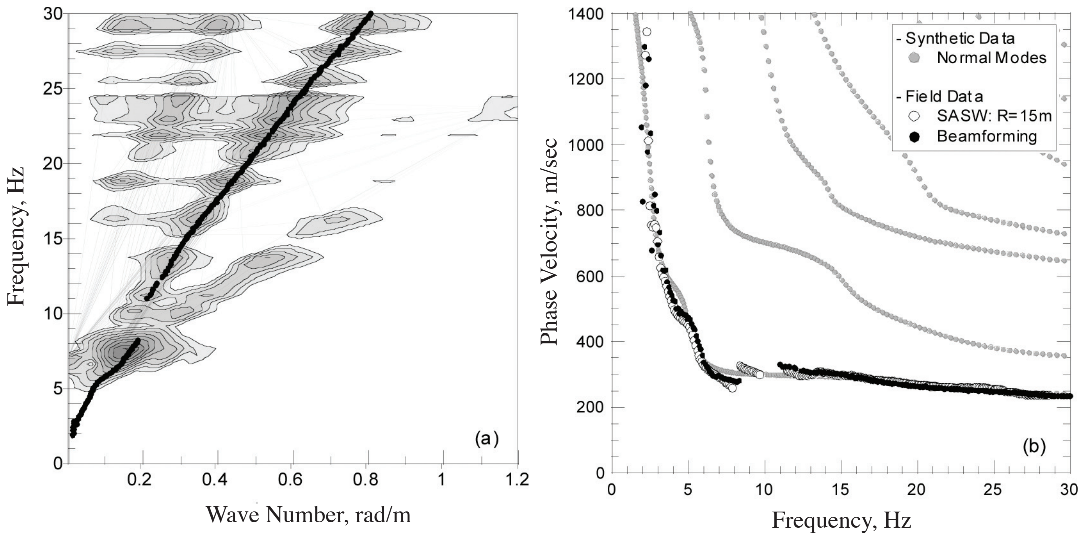

FIGURE 5. Results of beamforming tests at a natural geotechnical site near Hornsby, Texas: (a) f-k spectrum and (b) phase-velocity dispersion curve

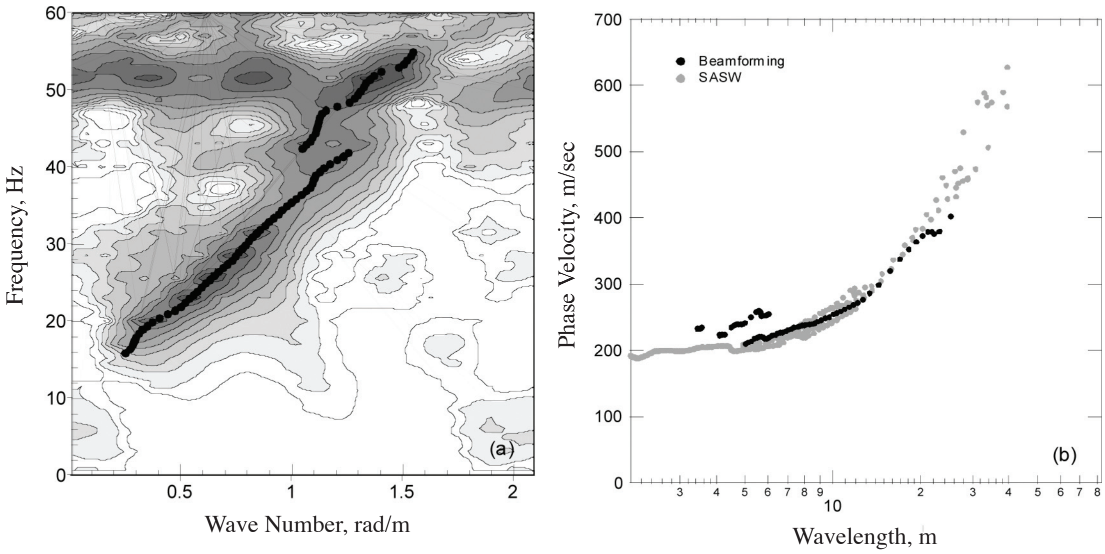

FIGURE 6. Results of beamforming tests a natural geotechnical site in DaeGu, Korea: (a) f-k spectrum and (b) phase-velocity dispersion curve

research are as follows: recovery of aliased wave numbers by the proposed algorithm was verified to be valid through numerical simulation of beamforming tests based on synthetic waveforms for typical layering systems and phase velocities determined by beamforming analysis proved to be apparent velocities, which superpose all the normal modes of surface waves. These surface-wave velocities are comparable with velocities determined by the SASW method.

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